



TITLE:

Study on Elastic Strain of the Ground in Earth Tides

AUTHOR(S):

OZAWA, Izuo

CITATION:

OZAWA, Izuo. Study on Elastic Strain of the Ground in Earth Tides.
Bulletins - Disaster Prevention Research Institute, Kyoto University 1957,
15: 1-36

ISSUE DATE:

1957-03

URL:

<http://hdl.handle.net/2433/123668>

RIGHT:

DISASTER PREVENTION RESEARCH INSTITUTE

BULLETIN No. 15

MARCH, 1957

STUDY ON ELASTIC STRAIN OF THE GROUND
IN EARTH TIDES

BY

IZUO OZAWA

KYOTO UNIVERSITY, KYOTO, JAPAN

DISASTER PREVENTION RESEARCH INSTITUTE
KYOTO UNIVERSITY
BULLETINS

Bulletin No. 15

March, 1957

Study on Elastic Strain of the Ground
in Earth Tides

By

Izuo OZAWA

Contents

	Page
1. Introduction	2
2. Fundamental theory	3
3. The influence of oceanic-tide on tidal strain	6
4. Observation	11
5. Comparison of observational results at two stations.....	17
6. Primary earth tidal strain	25
7. Summary	32
References	32
Appendix	35

Study on Elastic Strain of the Ground in Earth Tides

By

Izuo OZAWA

1. Introduction

Observation of the strain of the earth's surface caused by the tide-generating force by means of extensometer was initiated by K. Sassa [1], and the value of the constant l , usually called the Shida's number [2] was, at the first time, discussed separately from other constants, k and h (Love's numbers) [4]. E. Nishimura [3] obtained the value of $D=1+k-h=0.66$ from tiltmetric observation at Barin and the value of $L=1+k-l=1.20$ by analysing tidal latitude variations at six stations of the International Latitude Service. The Love's number k can be obtained from observation of the Chandler's period [5] of latitude variations, and according to Pollack [6], [7] it is estimated to be $k=0.287$. If the earth is considered homogenous and incompressible, the relations of $l=3/10h$ [2] and $k=3/5h$ [4] exist theoretically. These relations do not concord with the practical observations of the earth tide obtained by means of tiltmeters, gravimeters and others.

The problem of determining the strain of gravitating elastic sphere, under the action of small disturbing force having a spherical harmonic potential, has been solved by Kelvin and Tait [34], A.E.H. Love [8] and others for the case in which the density and elastic moduli are uniform throughout the body. Thereafter L. M. Hoskins [9] has also solved the problem in a case where the density and the elastic moduli are functions of distance from the earth's center. Recently, H. Takeuchi [10] calculated tidal deformation of the earth with internal constitutions inferred from recent seismological works [11], [12] combined with K. E. Bullen's [13] model of density distribution within the earth.

The tidal strain of the earth crust due to the attraction of heavenly bodies includes the so-called oceanic effect or load-tide which is the strain due to tidal variation of the load of sea water besides the so-called direct effect or bodily tide which is the deformation under the direct influence of the tide-generating force.

The direct effect means the tidal deformation which is assumed to occur if there were no ocean on the earth. The investigation of this oceanic effect is, needless to say, necessary and important in order not only to study the direct effect of the earth tide, but also to investigate the nature and structure of the earth crust by applying the load effect of sea water to the earth surface. In the present paper, the results obtained from the observations of the earth's tidal strain by using various types of extensometers at three stations are presented. The three stations concerned are Osakayama in Shiga Prefecture, 65 km distant from the nearest sea, Kishu Mine in Mie Prefecture, 15 km distant from the nearest sea, and Suhara in Wakayama Prefecture, close by the sea. From the observational results, the strain of ground caused by the tidal load of sea-water is discussed in some detail. Further, the tidal extensions in individual six directions at Osakayama Observatory are analysed and the vertical strain, horizontal areal strain and cubical dilatation in direct effect of the earth tides are also calculated.

2. Fundamental theory

Assuming that the tide-generating potential W_2 is proportional to $S_2 r^2$, where S_2 is a spherical surface harmonic of order 2 and a simple harmonic function of the time, and r is the radius vector from the earth's center. The components of radial, meridional and prime vertical displacements, u_r , u_θ and u_ϕ due to the tide-generating potential W_2 are written as follows :

$$\left. \begin{aligned} u_r &= \frac{2F(r) + G(r)r^2}{r} W_2 \\ u_\theta &= \frac{F(r)}{r} \frac{\partial W_2}{\partial \theta} \\ u_\phi &= \frac{F(r)}{r \sin \theta} \frac{\partial W_2}{\partial \phi} \end{aligned} \right\} \dots\dots\dots (1),$$

where $F(r)$ and $G(r)$ are function of r only, θ is colatitude, and ϕ is longitude.

In polar coordinate r, θ, ϕ , the following formulae will obtain : for the strains,

$$\left. \begin{aligned} e_{rr} &= \frac{\partial u_r}{\partial r}, \quad e_{\theta\theta} = \frac{1}{r} \frac{\partial u_r}{\partial \theta} + \frac{u_r}{r}, \quad e_{\phi\phi} = \frac{1}{r \sin \theta} \frac{\partial u_\phi}{\partial \phi} + \frac{u_\phi}{r} \cot \theta + \frac{u_r}{r} \\ e_{\phi\phi} &= \frac{1}{r} \left(\frac{\partial u_\phi}{\partial \theta} - u_\phi \cot \theta \right) + \frac{1}{r \sin \theta} \frac{\partial u_\theta}{\partial \phi}, \quad e_{\phi r} = \frac{1}{r \sin \theta} \frac{\partial u_r}{\partial \phi} + \frac{\partial u_\phi}{\partial r} - \frac{u_\phi}{r} \end{aligned} \right\} (2),$$

$$e_{r\theta} = \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r} + \frac{1}{r} \frac{\partial u_r}{\partial \theta}$$

for the cubical dilatation

$$\Delta = \frac{1}{r^2 \sin \theta} \left\{ \frac{\partial}{\partial r} (r^2 u_r \sin \theta) + \frac{\partial}{\partial \theta} (r u_\theta \sin \theta) + \frac{\partial}{\partial \phi} (r u_\phi) \right\}.$$

Using (1), (2) and the partial differential equation

$$\frac{1}{\sin^2 \theta} \frac{\partial^2 S_2}{\partial \phi^2} + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial S_2}{\partial \theta} \right) + 6S_2 = 0 \quad \dots\dots\dots(3),$$

the components of strain due to the disturbing potential W_2 may be written as follows:

for the strains,

$$\left. \begin{aligned} e_{rr} &= \frac{1}{r^2} \left\{ 2r \frac{dF(r)}{dr} + 2F(r) + r^3 \frac{dG(r)}{dr} + 3r^2 G(r) \right\} W_2 \\ e_{\phi\phi} &= \frac{1}{r^2} \left[\frac{F(r)}{\sin^2 \theta} \frac{\partial^2 W_2}{\partial \phi^2} + F(r) \cot \theta \frac{\partial W_2}{\partial \theta} + \{2F(r) + r^2 G(r)\} W_2 \right] \\ e_{\theta\theta} &= \frac{1}{r^2} \left[F(r) \frac{\partial^2 W_2}{\partial \theta^2} + \{2F(r) + r^2 G(r)\} W_2 \right] \\ e_{r\phi} &= \frac{1}{r^2 \sin \theta} \left\{ r \frac{dF(r)}{dr} + r^2 G(r) + 2F(r) \right\} \frac{\partial W_2}{\partial \phi} \\ e_{r\theta} &= \frac{1}{r^2} \left\{ r \frac{dF(r)}{dr} + r^2 G(r) + 2F(r) \right\} \frac{\partial W_2}{\partial \theta} \\ e_{\theta\phi} &= \frac{F(r)}{r^2 \sin \theta} \left\{ 2 \frac{\partial^2 W_2}{\partial \theta \partial \phi} - \cot \theta \frac{\partial W_2}{\partial \phi} \right\} \end{aligned} \right\} \dots\dots(4-1),$$

for the horizontal areal strain,

$$\Sigma = e_{\theta\theta} + e_{\phi\phi} = \frac{2}{r^2} \{r^2 G(r) - F(r)\} W_2 \quad \dots\dots\dots(4-2),$$

for the cubical dilatation

$$\Delta = \left\{ \frac{2}{r} \frac{dF(r)}{dr} + r \frac{dG(r)}{dr} + 5G(r) \right\} W_2 \quad \dots\dots\dots(4-3).$$

And potential and strain components of component tide are as follows:

1. Semi-diurnal tide at $r=a$ (a is the radius of the earth).

For the potential of tide-generating force,

$$W_2 \left(\frac{1}{2} \right) = a^2 A_2 \sin^2 \theta \cos 2(t_2 + \phi) \quad \dots\dots\dots(5-1),$$

where t_2 is the hour angle of the heavenly body at Greenwich and A_2 is a factor containing declination of the heavenly body.

For the strains,

$$\left. \begin{aligned}
 e_{rr} &= A_2 \left\{ 2F(a) + 2a \frac{dF(a)}{dr} + 3a^2 G(a) + a^3 \frac{dG(a)}{dr} \right\} \sin^2 \theta \cos 2(t_2 + \phi) \\
 e_{\theta\theta} &= A_2 \{ 2F(a) (\cos 2\theta + \sin^2 \theta) + a^2 G(a) \sin^2 \theta \} \cos 2(t_2 + \phi) \\
 e_{\phi\phi} &= A_2 \{ -2F(a) + a^2 G(a) \sin^2 \theta \} \cos 2(t_2 + \phi) \\
 e_{\theta\phi} &= A_2 (-6 \cos \theta) F(a) \sin 2(t_2 + \phi) \\
 e_{r\theta} &= e_{r\phi} = 0 \\
 \Sigma &= e_{\theta\theta} + e_{\phi\phi} = 2A_2 \{ a^2 G(a) - F(a) \} \sin^2 \theta \cos 2(t_2 + \phi) \\
 e_{\theta\theta} - e_{\phi\phi} &= 2A_2 F(a) (\cos 2\theta + \sin^2 \theta + 1) \cos 2(t_2 + \phi) \\
 \Delta &= \left\{ 2a \frac{dF(a)}{dr} + a^3 \frac{dG(a)}{dr} + 5a^2 G(a) \right\} A_2 \sin^2 \theta \cos 2(t_2 + \phi)
 \end{aligned} \right\} (5-2).$$

2. Diurnal tide at $r=a$.

For the potential of tide-generating force,

$$W_2(1) = a^2 A_1 \sin 2\theta \cos(t_1 + \phi) \dots\dots\dots (6-1),$$

where A_1 is another factor containing declination of the heavenly body.

For the strains,

$$\left. \begin{aligned}
 e_{rr} &= A_1 \left\{ 2F(a) + 2a \frac{dF(a)}{dr} + 3a^2 G(a) + a^3 \frac{dG(a)}{dr} \right\} \sin 2\theta \cos(t_1 + \phi) \\
 e_{\theta\theta} &= A_1 \{ a^2 G(a) - 2F(a) \} \sin 2\theta \cos(t_1 + \phi) \\
 e_{\phi\phi} &= A_1 \{ a^2 G(a) \} \sin 2\theta \cos(t_1 + \phi) \\
 e_{\theta\phi} &= A_1 F(a) \{ 4 - 2 \cot^2 \theta \} \sin(t_1 + \phi) \\
 e_{r\phi} &= e_{r\theta} = 0 \\
 \Sigma &= e_{\theta\theta} + e_{\phi\phi} = 2A_1 \{ a^2 G(a) - F(a) \} \sin 2\theta \cos(t_1 + \phi) \\
 e_{\theta\theta} - e_{\phi\phi} &= -2F(a) A_1 \sin 2\theta \cos(t_1 + \phi) \\
 \Delta &= \left\{ 2a \frac{dF(a)}{dr} + a^3 \frac{dG(a)}{dr} + 5a^2 G(a) \right\} A_1 \sin 2\theta \cos(t_1 + \phi)
 \end{aligned} \right\} (6-2).$$

According to formulae (5) and (6), the strains $e_{\theta\phi}$ and $e_{\theta\theta} - e_{\phi\phi}$ are function of $F(r)$ only, and according to formulae (6), $e_{\phi\phi}$ is a function of $G(r)$ only.

Introducing the following notations

$$\left. \begin{aligned}
 \frac{H(r)}{g(r)} &= \frac{2F(r) + r^2 G(r)}{r} \\
 \frac{L(r)}{g(r)} &= \frac{F(r)}{r} \\
 h &= H(a) \\
 l &= L(a)
 \end{aligned} \right\} \dots\dots\dots (7),$$

where g is the undisturbed gravity at r , we get for horizontal areal strain $\Sigma(a)$,

$$\Sigma(a) = e_{\theta\theta}(a) + e_{\phi\phi}(a) = 2(h-3l) \frac{W_2}{ag} \quad \dots\dots\dots(8).$$

Since normal and tangential stress must vanish on the earth surface, we get

$$\left. \begin{aligned} \widehat{rr} &= \lambda \Delta(a) + 2\mu e_{rr}(a) = 0 \\ \widehat{r\theta} &= \mu e_{r\theta}(a) = 0 \\ \widehat{r\phi} &= \mu e_{r\phi}(a) = 0 \end{aligned} \right\} \quad \dots\dots\dots(9),$$

where λ and μ are Lamé's elastic constants.

The extension E_i in any direction i will be given by

$$E_i = e_{rr}a^2 + e_{\theta\theta}\beta^2 + e_{\phi\phi}\gamma^2 + e_{\theta\phi}\beta\gamma \quad \dots\dots\dots(10),$$

where a , β and γ denote the direction cosines of the direction i taken to radius, meridian and prime vertical circles respectively. And we are able to obtain the components of the strain from relation (10) by means of observation of E_i in more than four different directions.

3. The influence of oceanic tide on tidal strain*

The results obtained from the earth tidal observation at a station near the sea contain, by any means, the large amount of ground deformation caused by the oceanic tide of neighboring sea, as well as the deformation caused by the direct effect of the sun and the moon. The oceanic effect of tidal tilt was studied by T. Shida [14], K. Sekiguchi [15]. D. Nukiyama, R. Takahashi [16], E. Nishimura [17], R. H. Corkan [18] and others [19], T. Hagiwara [20] and his group carried out the observation of tidal extensions at Aburatsubo close by the sea (20 m distant from the sea-side) and studied the influence of oceanic tide in a case where the sea bed is inclined at the coast. The present author [21] observed tidal strains at Osakayama, and compared the results of his observation with that at Makimine, discussing the oceanic effect at the stations, both considerably distant from the sea.

Theoretically the problem is concerned with the so-called "Boussinesq's

* cf. Appendix

Problem'' which was first discussed by J. Boussinesq [22], with regard to the point load problem on semi-infinite elastic body, and since then it has been treated by many researchers such as K. Terazawa [23], H. Nagaoka [24], H. Lamb [25], A.E.H. Love [26], K. Sezawa and G. Nishimura [27]. M. Matsu-mura [28], [29] and G. Nishimura [30] has obtained the surface deformation of semi-infinite elastic solid with surface layer due to any loading.

Let us suppose that the earth consists of uniform semi-infinite elastic medium. Let x and y axes be horizontal along the earth's surface, and z axis vertical inward. When a force P due to a point load acts at the origin parallel to the z axis, the displacement u, v, w of x, y, z components are, according to J. Boussinesq, given by

$$\left. \begin{aligned} u &= \frac{P}{4\pi\mu} \frac{xz}{r^3} - \frac{P}{4\pi(\lambda+\mu)} \frac{x}{r(z+r)} \\ v &= \frac{P}{4\pi\mu} \frac{yz}{r^3} - \frac{P}{4\pi(\lambda+\mu)} \frac{y}{r(z+r)} \\ w &= \frac{P}{4\pi\mu} \frac{z^2}{r^3} + \frac{P(\lambda+2\mu)}{4\pi\mu(\lambda+\mu)} \frac{1}{r} \end{aligned} \right\} \dots\dots\dots(11),$$

where

$$r^2 = x^2 + y^2 + z^2$$

From the equation (11) the components of strain are given as follows.

$$\left. \begin{aligned} e'_{xx} &= \frac{\partial u'}{\partial x} = \frac{P}{4\pi\mu} \left(\frac{z}{r^3} - \frac{3x^2z}{r^5} \right) - \frac{P}{4\pi(\lambda+\mu)} \left\{ \frac{(r^2-x^2)(z+r)-rx^2}{r^3(z+r)^2} \right\} \\ e'_{yy} &= \frac{\partial v'}{\partial y} = \frac{P}{4\pi\mu} \left(\frac{z}{r^3} - \frac{3y^2z}{r^5} \right) - \frac{P}{4\pi(\lambda+\mu)} \left\{ \frac{(r^2-y^2)(z+r)-ry^2}{r^3(z+r)^2} \right\} \\ e'_{zz} &= \frac{\partial w'}{\partial z} = \frac{P}{4\pi\mu} \left(\frac{2z}{r^3} - \frac{3z^3}{r^5} \right) - \frac{P(\lambda+2\mu)}{4\pi\mu(\lambda+\mu)} \frac{z}{r^3} \\ e'_{xy} &= \frac{\partial u'}{\partial y} + \frac{\partial v'}{\partial x} = -\frac{P}{4\pi\mu} \frac{3xyz}{r^5} + \frac{P}{4\pi(\lambda+\mu)} \frac{2xy(z+2r)}{r^3(z+r)^2} \\ e'_{xz} &= \frac{\partial w'}{\partial x} + \frac{\partial u'}{\partial z} = -\frac{P}{4\pi\mu} \frac{6xz^2}{r^5} \\ e'_{yz} &= \frac{\partial w'}{\partial y} + \frac{\partial v'}{\partial z} = -\frac{P}{4\pi\mu} \frac{6yz^2}{r^5} \\ \Delta' &= e'_{xx} + e'_{yy} + e'_{zz} = -\frac{P}{4\pi(\lambda+\mu)} \frac{2z}{r^3} \end{aligned} \right\} \dots(12),$$

where the sign “'” refers to the displacement and strain related with the oceanic effect. From (12), the components of strain at the surface except the

origin become as follows.

$$\left. \begin{aligned} (e'_{xx})_{z=0} &= \frac{P}{4\pi(\lambda+\mu)} \frac{x^2-y^2}{r^4} \\ (e'_{yy})_{z=0} &= \frac{P}{4\pi(\lambda+\mu)} \frac{y^2-x^2}{r^4} = -(e'_{xx})_{z=0} \\ (e'_{zz})_{z=0} &= 0 \\ (e'_{xy})_{z=0} &= \frac{P}{4\pi(\lambda+\mu)} \frac{4xy}{r^4} \\ \Sigma' &= (e'_{xx} + e'_{yy})_{z=0} = 0 \\ \Delta'_{z=0} &= 0 \end{aligned} \right\} \dots\dots\dots(13).$$

Let a new coordinate x' , y' , z' be parallel to the former coordinate x , y , z and its origin be removed from the load point P to an observing point O , and let r and ψ be the length of the vector OP and the angle between x' -axis and OP respectively. Then the equations (13) will become

$$\left. \begin{aligned} (e'_{x'x'})_{z'=0} &= \frac{P}{4\pi(\lambda+\mu)} \frac{\cos 2\psi}{r^2} \\ (e'_{y'y'})_{z'=0} &= -\frac{P}{4\pi(\lambda+\mu)} \frac{\cos 2\psi}{r^2} \\ (e'_{x'y'})_{z'=0} &= \frac{P}{4\pi(\lambda+\mu)} \frac{2 \sin 2\psi}{r^2} \\ (e'_{z'z'})_{z'=0} &= (e'_{y'y'})_{z'=0} = (e'_{x'x'})_{z'=0} = (e'_{x'y'} + e'_{y'y'})_{z'=0} = (\Delta')_{z'=0} = 0 \end{aligned} \right\} (14).$$

where

$$\begin{aligned} x' &= -r \cos \psi \\ y' &= -r \sin \psi \\ z' &= z. \end{aligned}$$

When each equation of (14) is integrated over the surface load, surrounded by concentric circles which have the radii r_n and r_{n+1} respectively, and whose center is the point of observation, the movable radii making polar angles ψ_n and ψ_{n+1} with x' -axis, the components of strain will be expressed by

$$\left. \begin{aligned} (e'_{x'x'})_{z'=0} &= \frac{P}{4\pi(\lambda+\mu)} \int_{r_n}^{r_{n+1}} \int_{\psi_n}^{\psi_{n+1}} \frac{\cos 2\psi}{r^2} r \, dr \, d\psi \\ &= \frac{P}{4\pi(\lambda+\mu)} \log \frac{r_{n+1}}{r_n} \left(\frac{1}{2} \right) (\sin 2\psi_{n+1} - \sin 2\psi_n) \\ (e'_{y'y'})_{z'=0} &= \frac{P}{4\pi(\lambda+\mu)} \int_{r_n}^{r_{n+1}} \int_{\psi_n}^{\psi_{n+1}} \frac{-\cos 2\psi}{r^2} r \, dr \, d\psi \\ &= \frac{P}{4\pi(\lambda+\mu)} \log \frac{r_{n+1}}{r_n} \left(\frac{-1}{2} \right) (\sin 2\psi_{n+1} - \sin 2\psi_n) = -(e'_{x'x'})_{z'=0} \end{aligned} \right\}$$

$$\begin{aligned}
(e'_{x'y'})_{z'=0} &= \frac{P}{4\pi(\lambda+\mu)} \int_{r_n}^{r_{n+1}} \int_{\psi_n}^{\psi_{n+1}} \frac{2 \sin 2\psi}{r^2} r \, dr d\psi \\
&= \frac{P}{4\pi(\lambda+\mu)} \log \frac{r_{n+1}}{r_n} (-1) (\cos 2\psi_{n+1} - \cos 2\psi_n) \\
&= \pm \frac{P}{4\pi(\lambda+\mu)} \log \frac{r_{n+1}}{r_n} \left\{ \sin 2\left(\psi_{n+1} \mp \frac{\pi}{4}\right) - \sin 2\left(\psi_n \mp \frac{\pi}{4}\right) \right\} \\
(e'_{z'z'})_{z'=0} &= (e'_{x'x'})_{z'=0} = (e'_{y'y'})_{z'=0} = 0 \\
(\Delta')_{z'=0} &= (e'_{z'x'} + e'_{y'y'})_{z'=0} = 0
\end{aligned} \quad \left. \vphantom{\begin{aligned} (e'_{x'y'})_{z'=0} \\ = \frac{P}{4\pi(\lambda+\mu)} \log \frac{r_{n+1}}{r_n} (-1) (\cos 2\psi_{n+1} - \cos 2\psi_n) \\ = \pm \frac{P}{4\pi(\lambda+\mu)} \log \frac{r_{n+1}}{r_n} \left\{ \sin 2\left(\psi_{n+1} \mp \frac{\pi}{4}\right) - \sin 2\left(\psi_n \mp \frac{\pi}{4}\right) \right\} \\ (e'_{z'z'})_{z'=0} \\ (\Delta')_{z'=0} \end{aligned}} \right\} (15),$$

where P is a unit surface load and is equivalent to $\rho \Delta h g$, where ρ and Δh are density of sea water and tidal height respectively.

The extensions of $E(0^\circ)$ in the direction of x' -axis and $E(\pi/2)$ in the direction of y' -axis are equal to $(e'_{x'x'})_{z'=0}$, $(e'_{y'y'})_{z'=0}$ respectively. The extension of $E(\pi/4)$ in the direction of $\psi = \pi/4$ will be

$$E\left(\frac{\pi}{4}\right) = (e'_{x'y'})_{z'=0} \cos^2 \frac{\pi}{4} = \frac{1}{2} (e'_{x'y'})_{z'=0} \dots\dots\dots (16).$$

Therefore

$$(e'_{x'y'})_{z'=0} = 2E\left(\frac{\pi}{4}\right) \dots\dots\dots (17),$$

namely, the longitudinal component of strain $(e'_{x'y'})_{z'=0}$ is twice as large as the extension in the direction of $\psi = \pi/4$. Therefore we are able to calculate easily strain componts $(e'_{x'x'})_{z'=0}$, $(e'_{y'y'})_{z'=0}$ and $(e'_{x'y'})_{z'=0}$ due to any local surface loads.

Let r , θ , ϕ be radius, colatitude and east longitude of the earth and u'_r , u'_θ , u'_ϕ be components of displacement caused by load respectively. Since it is probable to suppose that the strain caused by oceanic tides has very short wave length compared with the radius of the earth a , it may be assumed,

$$\left. \begin{aligned} \left(\frac{u'_r}{a}, \frac{u'_\theta}{a}, \frac{u'_\phi}{a} \right) &\ll \left(\frac{1}{a} \frac{\partial u'_\theta}{\partial \theta}, \frac{1}{a \sin \theta} \frac{\partial u'_\phi}{\partial \phi} \right) \end{aligned} \right\} \dots\dots\dots (18).$$

and $(\theta, \phi) \ll 1$

From (18), a relation between strain componts with respect to both systems of coordinate (r, θ, ϕ) and (x', y', z') will be obtained as follows.

$$\left. \begin{aligned} e'_{\theta\theta} &= e'_{x'x'}, & e'_{\phi\phi} &= e'_{y'y'}, & e'_{r\phi} &= e'_{z'z'} \\ e'_{\theta\phi} &= -e'_{x'y'}, & e'_{r\theta} &= e'_{z'x'}, & e'_{r\phi} &= -e'_{y'z'} \end{aligned} \right\} \dots\dots\dots (19),$$

where x' , y' and z' axes are directed toward the north, the east and the

underground respectively.

In actual calculation of the oceanic effect $e'_{\theta\theta}$, $e'_{\phi\phi}$ and $e'_{\theta\phi}$, we prepare a transparent section paper sectioned with the graduation of concentric circles centering on the observational point and with the radial vector graduation in such a way as we shall have

$$\log \frac{r_2}{r_1} = C_1, \quad \sin 2\psi_2 - \sin 2\psi_1 = C_2$$

and put this section paper right on the tide chart, fitting its center on the observational spot at an appropriate rotation angle. By counting the number of sections involved within the sea area on the tide chart which we intend to calculate, the computation of the related equations (15) and (17) can easily be done.

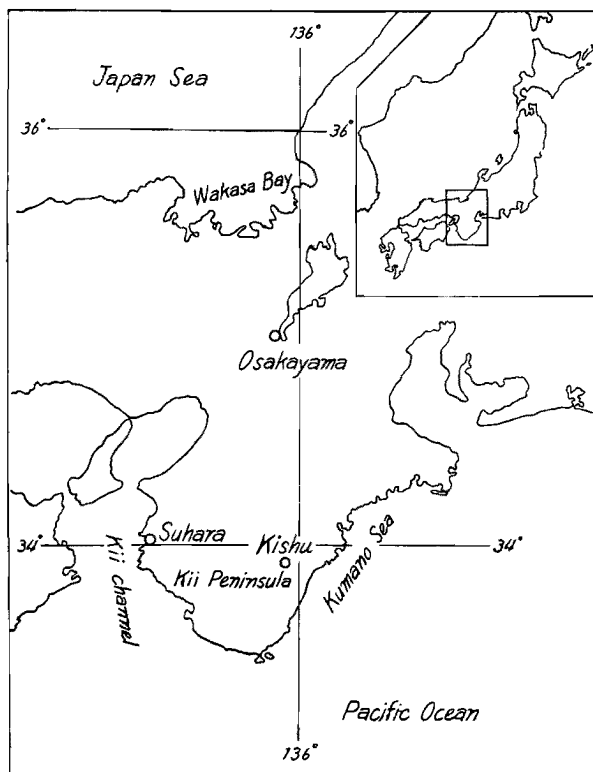


Fig. 1. Location of observatories

4. Observation

Observational stations concerned in tidal strain of the earth are Osakayama Observatory, Kishu Mine Observatory and Suhara Observatory, their locations being shown in Fig. 1.

1) Observation at Osakayama : Osakayama Observatory is situated at $34^{\circ}59'6''\text{N}$ and $135^{\circ}51'5''\text{E}$. Long. The nearest sea is the Wakasa Bay which is at a distance of 65 km from the observatory. The observing room is located in the middle of an abandoned railroad tunnel of 700 m length, 150 m under the earth's surface. No artificial disturbance preventing us from the observation is in neighborhood. The geological condition is of the claystone belonging to the Chichibu palaeozoic system. The change of temperature in the observing room is about 0.4°C in annual variation and smaller than 0.01°C in daily variation, and the amplitude of linear strain is smaller than 10^{-9} c.g.s. in daily variation. At this observatory, extensometers are placed in six directions three horizontal, two diagonal and one vertical directions. Besides, four additional extensometers are set up in order to make comparative observations of particular directions. The types and sensitivity of instruments are shown in Table 1.

The full informations on the instruments were given in previous reports [1], [31]. By the ordinary method of harmonic analysis the values of component tides of M_2 , S_2 , K_1 , O_1 and others were calculated. Further adjustment necessitated by differences in time length of running mean and the period of com-

Direction of observation	Type of instruments	Span of measured two points	Sensitivity in 10^{-8}c.g.s./mm
Azimuth $\text{N}38^{\circ}\text{E}$	Horizontal Sassa-type (wire)	20 m	0.41~0.63
Azimuth $\text{N}29^{\circ}\text{W}$	Horizontal Sassa-type (wire)	4.2 m	3.0 ~9.8
Azimuth $\text{N}61^{\circ}\text{E}$	Horizontal Spring-type (wire)	9.6 m	1.82
Vertical	Spring-type (wire)	4.4 m	1.85~2.24
Vertical	Pivot-type (wire)	4.0 m	3.1
Azimuth $\text{S}52^{\circ}\text{E}$ Elevation 45°	Rod-type	5.1 m	1.83
Azimuth $\text{N}38^{\circ}\text{E}$ Elevation 48°	Pivot-type (wire)	6.4 m	2.46

Table I.

Table III. Analysed values of component tide.

Division	Direction of observation	Epoch analysed	Sensitivity in 10^{-8} c.g.s./mm	M ₂ -component Ampli- Phase tide lag in 10^{-8}	S ₂ -component Ampli- Phase tide lag in 10^{-8}	K ₂ -component Ampli- Phase tide lag in 10^{-8}	K ₁ -component Ampli- Phase tide lag in 10^{-8}	O ₁ -component Ampli- Phase tide lag in 10^{-8}
1	N38°E	1947 Oct. 24.- 1949 Feb. 24.-	0.41 -0.63	0.33 43°				
1'	N38°E	1948 Feb. 24.- 1949 Feb. 28.-	0.41 -0.63		0.177 350.1°	0.084 22.3°	0.187 290.1°	0.123 358.4°
2	N29°W	1953 Feb. 19.- Mar. 19.-	9.80	1.98 23°	0.66 25°	0.18 25°		
3	N61°E	1953 Feb. 12.- Mar. 12.-	3.10	0.50 9°	0.22 72°	0.06 72°		
4	Vertical	1952 Dec. 25.- 1953 Jan. 23.-	2.24	0.44 215°				
5	Vertical	1953 Feb. 12.- Mar. 12.-	2.22	0.63 201°				
6	Vertical	1953 Mar. 12.- Apr. 10.-	2.22	0.64 193°				
7	N29°W	1954 May 2.- May 31.-	3.00	1.38 8°	0.77 346°	0.21 346°	0.86 18°	1.19 34°
8	N61°E	1954 Aug. 31.- Sep. 29.-	1.82	0.56 14°	0.13 30°	0.04 30°	0.18 359°	0.36 29°
9	Vertical	1954 Oct. 7.- Nov. 5.-	1.85	0.74 174°	0.65 158°	0.16 158°	0.73 187°	0.61 232°
10	Vertical	1954 Nov. 5.- Dec. 4.-	1.85	0.84 192°	0.39 142°	0.11 142°	0.62 193°	
11	Azimuth S52°E Elevation 45°	1955 Jan. 7.- Jan. 21.-	1.34	0.12 56°				
12	Azimuth S52°E Elevation 45°	1955 Sep. 20.- Oct. 19.-	1.34	0.06 271°				
13	Azimuth N38°E Elevation 48°	1956 Jul. 13.- Aug. 11.-	2.46	0.26 331°				

ponent tides is made at the final step as follows.

Component tide	Period of component tide in hour	Time length of 24 hours	Running mean 25 hours
M_2	12.42	1.035	1.006
S_2	12.00	1.000	1.040
K_1	23.93	1.003	1.043
K_2	11.97	1.003	1.043
O_1	25.47	1.072	1.018

Table II. Coefficients of adjustment

The amplitude and phase of component tide thus obtained and the ratio of amplitude between component tides S_2/M_2 , K_1/M_2 and O_1/M_2 are shown in Table III and Table IV.

Division	Direction	S_2/M_2	K_1/M_2	O_1/M_2
2	N29°W	0.33		
3	N61°E	0.44		
7	N29°W	0.56	0.62	0.86
8	N61°E	0.23	0.32	0.64
9	Vertical	0.88	0.99	0.82
10	Vertical	0.92	0.74	
1'	N38°E	0.53	0.56	0.37

Table IV. Ratio of component tide.

2) Observation at Kishu Mine : Kishu Mine Observatory is situated at the point of 135°53.4'E Long. and 33°51.7'N. The observation point is about 15 km distant from the sea of Kumano which is on the east coast of the Kii peninsula. The observing room is located within the gallery at a curved distance of about 300 m from the mine gate on the ground, and about 100 m below the earth surface, and in the sand-stone of the tertiary system. The instruments used for the observation are shown in Table V.

Direction of observation	Type of instrument	Span of measured two points	Sensitivity in 10^{-8} c.g.s./mm	Epoch of observation
N79.5°W	Horizontal spring-type (wire)	9.00m	1.62	Mar. 1955
N60°E	Horizontal spring-type (wire)	7.53	1.27	Sep. 1955
N30°W	Horizontal spring-type (wire)	4.54	1.22	Sep. 1955

Table V Instruments used at Kishu Mine.

The component tides were obtained by harmonic analysis of the records of one month for each of three components in a same manner as was done in the case of Osakayama. The amplitude and phase of M_2 -term are shown in Table VI.

Azimuth of observation	Epoch analysed	Sensitivity in 10^{-8} c.g.s./mm	Amplitude in 10^{-8}	Phase lag (κ)
N79.5°W	Apr. 9~May 8, 1955	1.62	0.063	32.9°
N30°W	Feb. 2~Mar.2, 1956	1.22	1.271	331.1
N60°E	Feb. 2~Mar.2, 1956	1.27	0.710	44.2

Table VI. Observed values of M_2 -term at Kishu.

Using observed values of extensions in three directions, the three components of horizontal strain of M_2 -term were calculated (Table VII.)

3) Observation at Suhara : Suhara Observatory is situated on the sea shore and in an old copper mine at $135^{\circ}11.7'E$ Long. and $34^{\circ}02.6'N$. The extensometers are installed at a place 50-90 m distant from the sea shore, 30-70 m from the mine gate and 30-60 m below the ground-surface. Neighboring geology is of chlorite schist and breccia belonging to Chichibu palaeozoic system and mesozoic system, and the station is just at the south side of Central Median Dislocation Line traversing through the Kii peninsula and Sikoku. The extensometers concerned are shown in Table VIII.

Component	Amplitude in 10^{-8}	Phase-lag (κ)
$e_{\theta\theta}$	1.691	5.2°
$e_{\phi\phi}$	0.244	256.2
$e_{\theta\phi}$	1.477	258.4

Table VII. M_2 -term of horizontal strain components at Kishu

Azimuth of observation	Type of extensometer	Span of measured two points	Sensitivity in 10^{-8} c.g.s./mm	Epoch of observation
N	Horizontal spring-type (wire)	19.55m	3.2~ 4.6	Dec. 1955
E	Horizontal spring-type (wire)	8.82	2.1~ 4.0	Dec. 1955
S70°E	Horizontal spring-type (wire)	11.00	10.1~12.6	Dec. 1955

Table VIII. Instrument used at Suhara.

The M_2 -term of the extension was analysed in a same process as at Kishu and Osakayama. And amplitude and phase angle of M_2 -term are shown in Table IX.

Azimuth of observation	Epoch Analysed	Sensitivity in 10^{-8} c.g.s./mm	Amplitude in 10^{-8}	Phase lag (κ)
NS	Mar. 9~Apr. 8, 1956	3.2	1.940	49.9°
EW	Mar. 9~Apr. 8, 1956	2.1	1.876	36.4
S70°E	Mar. 19~Apr. 18, 1956	10.1	2.307	52.3

Table IX.

Using the above mentioned observed values, M_2 components of horizontal strains were obtained which are shown in Table X.

In Table XI, M_2 -terms of horizontal strain components $e_{\theta\theta}$, $e_{\phi\phi}$, $e_{\theta\phi}$ and horizontal areal strain at Osakayama, Kishu and Suhara are shown together.

From the results of Table XI, it is clearly seen that the amplitudes of horizontal areal strain are not so different at three observatories, and their phase angles are nearly 0°. The relation between amplitudes and phase angles of M_2 -term and their azimuth obtained from the observations are shown in Table XII and Figs. 2(a), (b), (c), (d). As is seen from Fig. 2, it appears that the amplitude of extension reaches its maximum in the azimuth of southeast and northwest and the amplitude reaches

Component	Amplitude in 10^{-8}	Phase-lag (κ)
$e_{\theta\theta}$	1.940	49.1°
$e_{\phi\phi}$	1.876	36.4
$e_{\theta\phi}$	2.092	96.0

Table. X. M_2 -terms of horizontal strain-component of obtained from observation at Suhara

Component	Osakayama		Kishu		Suhara	
	Amplitude in 10^{-8}	Phase lag (κ)	Amplitude in 10^{-8}	Phase lag (κ)	Amplitude in 10^{-8}	Phase lag (κ)
$e_{\theta\theta}$	0.957	29.6°	1.691	5.2°	1.940	49.1°
$e_{\phi\phi}$	1.315	5.5	0.244	256.2	1.876	36.4
$e_{\theta\phi}$	1.650	74.8	1.477	258.4	2.092	96.0
Horizontal areal strain	2.223	12.3	1.628	356.9	2.780	42.8

Table XI M_2 -term of horizontal strain component (observation)

Azimuth	Osakayama		Kishu		Suhara	
	Amplitude in 10^{-8}	phase lag (κ)	Amplitude in 10^{-8}	Phase lag (κ)	Amplitude in 10^{-8}	Phase lag (κ)
0°	0.724	22.6°	1.691	5.2°	1.940	49.1°
15	1.060	12.9	1.514	351.1	2.384	58.3
30	1.400	7.1	1.260	335.7	2.595	62.0
45	1.650	3.8	1.021	311.4	2.658	61.8
60	1.810	1.2	0.811	288.4	2.493	57.7
75	1.670	0.1	0.576	269.0	2.240	49.0
90	1.260	0.3	0.244	256.2	1.875	36.4
105	0.986	0.3	0.198	49.0	1.704	22.5
120	0.662	9.4	0.699	44.3	1.550	11.7
135	0.344	31.7	1.172	35.5	1.500	9.9
150	0.338	49.9	1.534	26.6	1.496	20.3
165	0.463	36.9	1.584	19.2	1.681	36.0
180	0.724	22.6	1.691	5.2	1.940	49.1

Table XII. Observed values : Relation between amplitudes and phase lag of M_2 -term and their azimuth.

its minimum in the direction of the right angle to it at Osakayama and Suhara. At Kishu mine, the amplitude reaches its maximum in the direction of north and south, and reaches its minimum in the direction of the right angle to it. In the three observatories, the phase angles (κ) of extension are positive, that is, cosine term is positive in all azimuth. The observ-

ed values of amplitude at Suhara are, as is expected, larger than those values at the two other stations.

5. Comparison of observational results at two stations

Now, let θ and ϕ be the co-latitude and longitude of observatory respectively. The potential of tide-generating force is given by

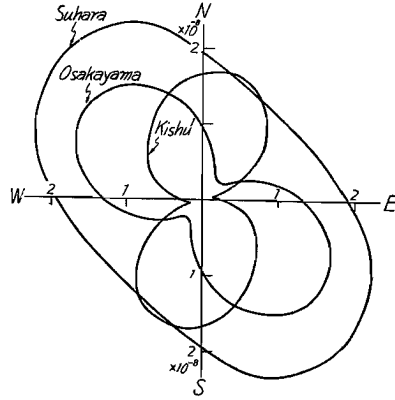


Fig. 2.(a) Relation between the amplitudes of M_2 -term and their azimuth obtained from the observations at Osakayama, Kishu and Suhara.

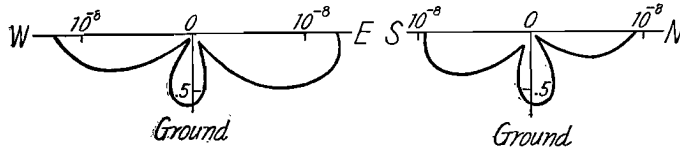


Fig. 2.(b) Relation between amplitudes of M_2 -term and their elevation obtained from the observation at Osakayama.

$$\begin{aligned} a^2 A_2 \sin^2 \theta \cos 2(t_2 + \phi) & \quad \text{for the semi-diurnal tide} \\ a^2 A_1 \sin 2\theta \cos(t_1 + \phi) & \quad \text{for the diurnal tide.} \end{aligned}$$

Since maximum difference of longitude between these three observatories is only $0^\circ 42'$ in angle and 2.8 minutes in the mean solar time, it may be assumed that their longitude are equal to each other. The difference between the tide-generating potential at any two observatories (θ_i, ϕ_i) and (θ_j, ϕ_j) will be

$$\begin{aligned} a^2 A_2 \{ \sin(\theta_i + \theta_j) \sin(\theta_i - \theta_j) \} \cos 2(t_2 + \phi) & \quad \text{for the semi-diurnal tide} \\ a^2 A_1 \{ 2 \cos(\theta_i + \theta_j) \sin(\theta_i - \theta_j) \} \cos(t_1 + \phi) & \quad \text{for the diurnal tide} \end{aligned}$$

and these numerical values are shown in Table XIII.

Since the greatest differences are only 1.8% in the semi-diurnal tide and

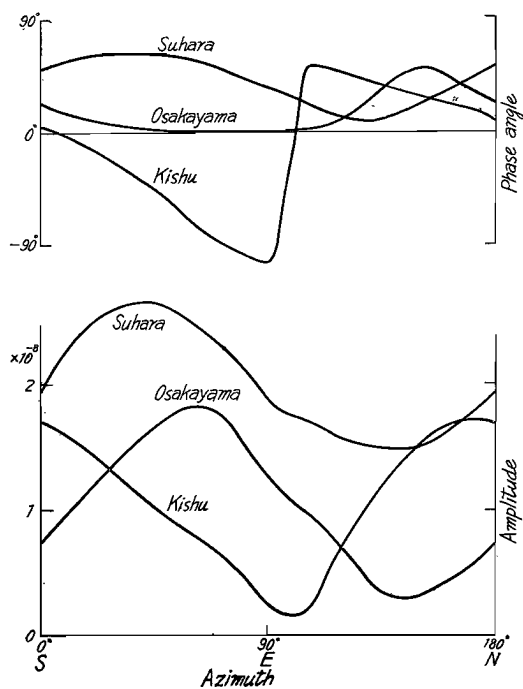


Fig. 2.(c) Relation between the amplitudes and phase angle of M_2 -term and their azimuth obtained from the observations.

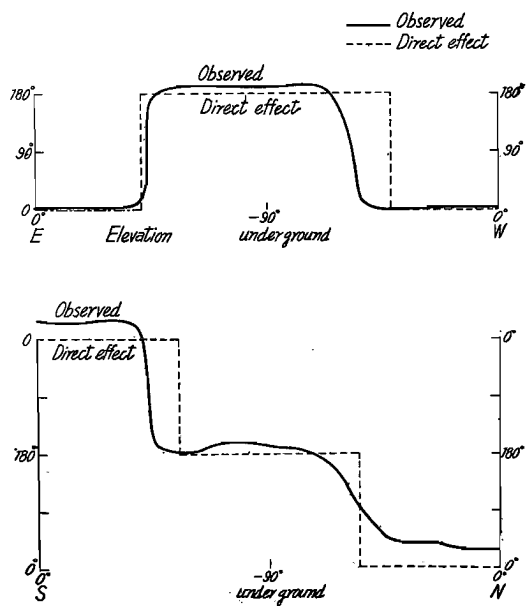


Fig. 2.(d) Relation between elevation and phase angle of linear strain of M_2 -term at Osakayama.

$$\times a^2 A_n \cos n (t_n + \phi)$$

	Osakayama-Kishu	Kishu-Suhara	Suhara-Osakayama
Semi-diurnal tide $n=2$	0.01840	-0.00286	-0.01548
Diurnal tide $n=1$	-0.01410	0.00232	0.01108

Table XIII. Differences between tide-generating potential at pairs of observatories.

pair of observatory com- ponent of strain	Osakayama-Kishu		Kishu-Suhara		Suhara-Osakayama	
	Amplitude in 10^{-8}	Phase angle (κ)	Amplitude in 10^{-8}	Phase angle (κ)	Amplitude in 10^{-8}	Phase angle (κ)
$e'_{\theta\theta}$	0.913	159.7°	1.402	288.0°	1.095	67.6°
$e'_{\phi\phi}$	1.414	14.9	2.051	221.2	1.003	79.8
$e'_{\theta\phi}$	2.653	45.2	3.532	181.0	2.389	139.4

Table XIV. The Differences between observed strain components at pairs of observatories.

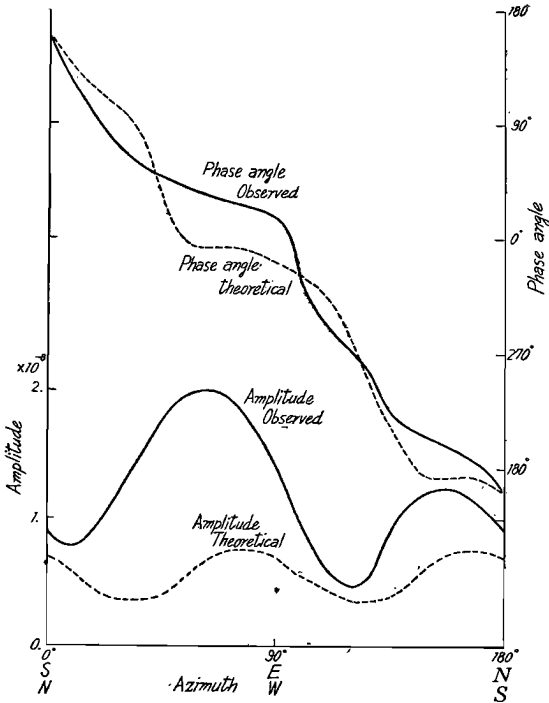


Fig. 3. The difference between the linear strains at pair of observatories.

(a) Osakayama-Kishu.

1.4 % in the diurnal tide, it is reasonably assumed in a first approximation that the direct effects of earth tide is nearly equal for three observational stations concerned. Accordingly, the direct effect of the earth tide is considered to be cancelled in the present treatment and the oceanic effect alone will be contained in the differences of these observed values. In Table XIV are shown the differences between the observed strain components at these three pairs of observatories ; Osakayama-

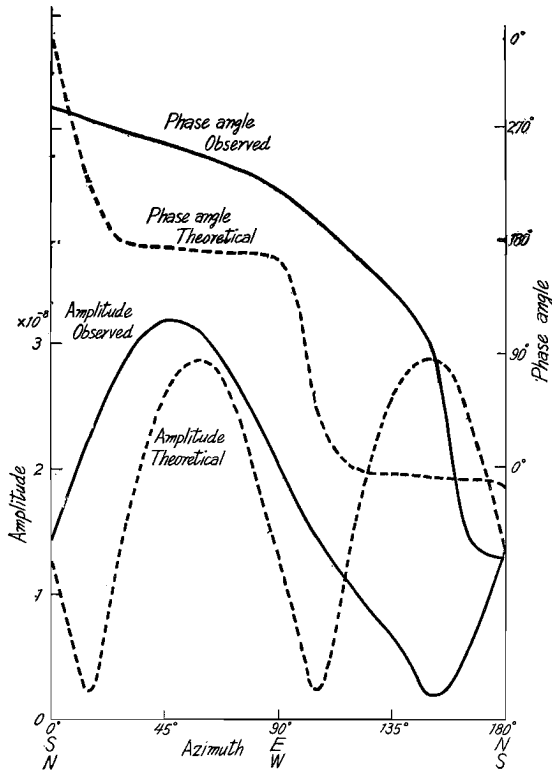


Fig. 3. The difference between the linear strain at pair of observatories.

(b) Kishu-Suhara

Kishu, Kishu-Suhara and Suhara-Osakayama.

Using these differences between components of strain at pairs of observatory, the relations of azimuth amplitude, and azimuth phase angle were calculated and shown in Table XV and are illustrated in Figs. 3. (a), (b), (c), (d), (e).

It is to be remarked that the azimuth in the table is reckoned from the south in counter-clockwise direction.

On the other hand, applying the M_2 co-tidal chart [32] and $(M_2 + S_2)$ co-range chart of the adjacent sea of Japan to

Pair of observatory Azimuth	Osakayama-Kishu		Kishu-Suhara		Suhara-Osakayama	
	Amplitude in 10^{-8}	Phase angle (κ)	Amplitude in 10^{-8}	Phase angle (κ)	Amplitude in 10^{-8}	Phase angle (κ)
0°	0.913	159.7°	1.402	288.0°	1.095	67.6°
15	0.837	105.5	2.234	278.0	1.392	91.8
30	1.266	66.1	2.868	268.0	1.731	104.2
45	1.756	47.3	3.164	259.3	1.848	108.8
60	1.994	35.5	3.072	249.5	1.753	108.0
75	1.855	26.4	2.649	237.6	1.401	100.6
90	1.414	14.9	2.051	221.2	1.003	79.8
105	0.755	304.9	1.468	198.5	0.883	43.1
120	0.476	270.0	1.037	170.0	1.070	17.6
135	0.900	221.1	0.674	141.0	1.222	10.5
150	1.204	186.8	0.194	99.3	1.188	16.3
165	1.177	186.8	0.550	305.2	1.060	35.6
180	0.913	159.7	1.402	288.0	1.095	67.6

Table XV. Relation between the difference of linear strain and its azimuth obtained at pair of observatories.

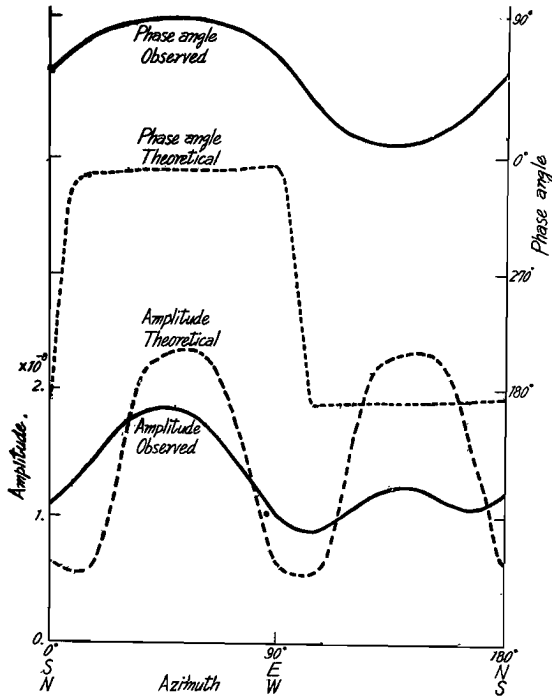


Fig. 3. The difference between the linear strains at pair of observatories.

(c) Suhara-Osakayama.

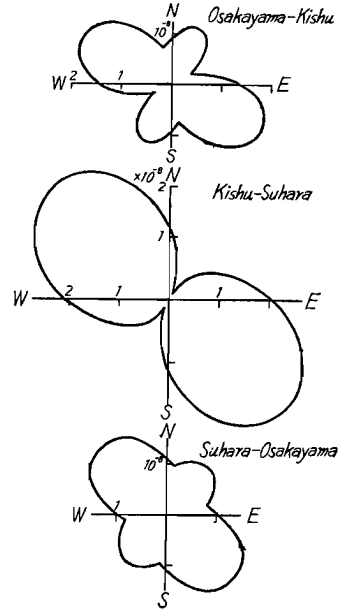


Fig. 3(d) Relation between differences of observed strain amplitude and their azimuth at pair of observatories.

each observatory, strain due to the tidal load of sea water on the surface of semi-infinite elastic body was calculated by means of above mentioned formula (15) for each observatory. The calculations were made for oceanic tide in concentric rings bounded by concentric circles, 0-100 km, 100-200 km,.....700 km-800 km around each observation point as their center and the calculated components of strain are shown in Table XVI where strains caused by load represent M_2 -terms. Then, the differences of the components of Table XVI among three observatories : Osakayama-Kishu, Kishu-Suhara and Suhara-Osakayama, were taken. They are listed in Table XVII. The relations between the azimuth-amplitude or the azimuth-phase angle of relative extension of oceanic effect for the couples of Osakayama-Kishu, Kishu-Suhara and Suhara-Osakayama, are shown in Table XVIII and Fig. 3, in which the furthestmost distance of sea area used for calculation is 800 km.

Let us compare the observational and theoretical relation between exten-

Table XVI. Oceanic-effect obtained by theoretical calculation.

$$c = \frac{2(M_2 + S_2)}{M_2} \bigg/ \frac{\rho g}{4\pi(\lambda + \mu)}, \text{ where } M_2 \text{ and } S_2 \text{ represent the } M_2 \text{ and } S_2 \text{ ranges respectively}$$

Radius of sea region	Osakayama				Kishu				Suhara			
	$ce'_{\theta\theta} (= -ce'_{\phi\phi})$		$ce'_{\theta\phi}$		$ce'_{\theta\phi} (= -ce'_{\phi\phi})$		$ce'_{\theta\phi}$		$ce'_{\theta\phi} (= -ce'_{\phi\phi})$		$ce'_{\theta\phi}$	
	$\times \cos 2t$	$\times \sin 2t$	$\times \cos 2t$	$\times \sin 2t$	$\times \cos 2t$	$\times \sin 2t$	$\times \cos 2t$	$\times \sin 2t$	$\times \cos 2t$	$\times \sin 2t$	$\times \cos 2t$	$\times \sin 2t$
0-100 k m	7.80	-2.57	-12.74	20.00	32.15	-11.09	-97.02	18.98	-2.46	-6.39	374.90	-20.80
100-200 k m	-6.15	7.53	-9.18	30.28	15.53	-2.72	-10.31	0.02	-43.32	5.91	7.78	7.40
200-300 k m	-13.56	5.16	-17.67	28.42	-3.78	6.12	-26.11	27.08	-10.21	-1.81	6.45	13.38
300-400 k m	-7.55	4.32	-5.71	30.50	0.74	4.24	14.84	12.34	-5.35	-10.19	9.97	13.54
400-500 k m	-4.29	1.59	2.60	14.21	-3.72	-2.06	6.34	10.12	-13.30	-3.58	2.78	7.89
500-600 k m	-3.03	1.03	1.55	9.82	-2.64	-0.60	1.77	9.11	-12.57	15.78	3.58	12.83
600-700 k m	-3.48	1.78	5.15	13.59	-2.22	-4.10	7.31	14.66	-4.24	8.97	8.26	16.21
700-800 k m	-3.40	1.22	10.24	5.86	-3.29	-0.07	10.29	6.97	-5.94	12.54	4.10	12.35

Table XVII. Difference of oceanic-effect at pair observatories obtained by theoretical calculation.

$$c = \frac{2(M_2 + S_2)}{M_2} \bigg/ \frac{\rho g}{4\pi(\lambda + \mu)}$$

Radius of sea region	Osakayama-Kishu				Kishu-Suhara				Suhara-Osakayama			
	$ce'_{\theta\theta} (= -ce'_{\phi\phi})$		$ce'_{\theta\phi}$		$ce'_{\theta\phi} (= -ce'_{\phi\phi})$		$ce'_{\theta\phi}$		$ce'_{\theta\phi} (= -ce'_{\phi\phi})$		$ce'_{\theta\phi}$	
	$\times \cos 2t$	$\times \sin 2t$	$\times \cos 2t$	$\times \sin 2t$	$\times \cos 2t$	$\times \sin 2t$	$\times \cos 2t$	$\times \sin 2t$	$\times \cos 2t$	$\times \sin 2t$	$\times \cos 2t$	$\times \sin 2t$
0-100 k m	24.35	8.52	84.28	1.02	34.61	-4.70	-471.92	39.78	-10.26	-3.82	387.64	-40.80
100-200 k m	-21.60	10.25	1.13	30.24	58.85	-8.63	-18.09	-7.38	-37.17	-1.62	16.96	-22.88
200-300 k m	-9.78	-0.96	8.44	1.34	6.34	7.93	-32.56	13.70	3.44	6.97	24.12	-15.04
300-400 k m	-8.29	0.08	-20.55	8.16	6.09	14.43	-4.87	-1.20	2.20	-14.51	15.68	-6.96
400-500 k m	-0.57	0.47	-3.74	4.09	9.58	1.47	3.56	2.23	-9.01	-1.94	0.18	-6.32
500-600 k m	-0.39	-0.43	-0.22	0.71	9.93	-16.38	-1.81	-3.72	-9.54	16.81	2.03	3.01
600-700 k m	-1.26	5.88	-2.16	-1.07	2.02	-13.07	-0.95	-1.55	-0.76	7.19	3.11	2.62
700-800 k m	-0.11	-1.15	-0.05	-1.08	2.65	-12.61	6.19	-5.38	-2.54	13.76	-6.14	6.46
0-800 k m	-66.43	22.66	67.09	43.43	130.07	-31.56	-507.04	36.48	-63.64	8.40	443.58	-79.91

sion and its azimuth with each other. In case of Osakayama-Kishu, the am-

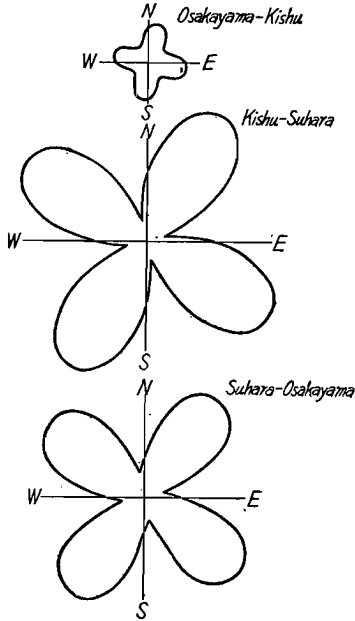


Fig. 3(e) Relation between theoretical relative strain amplitude and its azimuth at pair of observatories.

plitude of extension reaches its maximum in the azimuth of N12°E-S12°W, and N78°W-S78°E in the calculated values by means of elastic theory, and both maximum amplitudes are equal. In the observed relation a greater maximum of amplitude of the linear strain is found in the direction of N65°W-S65°E, and a smaller maximum of amplitude in the direction of N 25°E-S25°W. The ratio of the amplitude of greater maximum to that of smaller one is 1.6 and the phase angles are 32° and 198° respectively, and the phase difference is nearly 180° in the directions at right angles to each other. The relations obtained by observation are in a good agreement with theoretical ones obtained by elastic theory in the pair of Osakayama and Kishu. In case of Kishu-Suhara, the calculated

Azimuth	Osakayama-Kishu		Kishu-Suhara		Suhara-Osakayama	
	Amplitude × c	Phase angle (κ)	Amplitude × c	Phase angle (κ)	Amplitude × c	Phase angle (κ)
0°	70	161.1°	134	346°	64	173°
15	50	143.1	23	232	57	374
30	36	97.9	155	180	163	349
45	39	32.9	254	176	225	350
60	63	353.2	286	173	227	350
75	75	353.3	242	171	165	353
90	70	341.1	134	166	64	353
105	50	323.1	23	52	57	194
120	36	277.9	155	0	163	169
135	39	212.9	254	356	225	170
150	63	173.2	286	353	227	170
165	75	173.3	242	351	165	173
180	70	161.1	134	346	64	173

Table XVIII Theoretical difference of oceanic effect at pair observatories. Relation between linear strain and its azimuth (calculation).

$$c = \frac{2(M_2 + S_2)}{M_2} \bigg/ \frac{\rho g}{4\pi(\lambda + \mu)}$$

amplitudes reach their maximum in the azimuth of $N32^{\circ}E-S32^{\circ}W$ and $N58^{\circ}W-S58^{\circ}E$, and the amplitudes are equal at their maximum, and the difference of phase angle is 180° . In the observation, there is a greater maximum in the azimuth of $N45^{\circ}W-S45^{\circ}E$, and smaller one is not remarkable. But the derivative of phase angle of the linear strain with respect to azimuth obtained by observation is parallel to the derivative derived by means of theoretical calculation. In the case of Suhara-Osakyama, the calculated values of amplitude reaches maximum in the direction of $N36^{\circ}E-S36^{\circ}W$ and in the direction of its right angle, and the maximum amplitudes in both directions are equal. In the observed values the amplitude reaches maximum in the azimuth of $N47^{\circ}W-S47^{\circ}E$ and a smaller maximum in the azimuth of its right angle and the ratio of amplitudes in the two maximum azimuths is 1.7 and the phase are 109° and 10° respectively: the amplitudes are not exactly equal in the orthogonal directions, but their phase-difference is 99° . The phase angle obtained by observations is not equal to theoretical one but the azimuth of maximum amplitude, and the derivative of phase angle with respect to azimuth are equal to each other. From these results, it is naturally assumed that the horizontal areal strain is nearly equal to 0 by tidal load of distant sea water. It may be considered that the elastic moduli of the earth is not homogeneous in the depth, but in the difference of strain at the pair of Osakayama-Kishu, the crust showed deformation of such a kind as if it is a homogeneous elastic body which had been theoretically treated by J. Boussinesq and others. The ratios of the amplitude in azimuths of maximum extensions (linear strain) obtained from observations to the corresponding amplitude obtained from theoretical calculations were computed and shown in Table XIX. The numerical values in Table XIX are in an arbitrary unit.

Pair of observatory	Osakayama-Kishu	Kishu-Suhara	Suhara-Osakayama
Ratio	2.08	2.22	1.60

Table XIX. Ratio of amplitude observed to that calculated in the azimuth of maximum linear strain.

The shortest distances from each observational station to the nearest sea are 65 km from Osakayama to the Wakasa Bay (Japan Sea), 15 km from Kishu to the Kumano Sea (the Pacific Ocean) and 60 m from Suhara to the Kii Channel. The distances between observational stations are 122 km

			Osakayama- Kishu	Kishu-Suhara	Suhara- Osakayama
Azimuth of maximum extension	greater	Calculation	N78°W	N58°W	N54°W
		Observation	N65°W	N49°W	N47°W
	smaller	Calculation	N12°W	N32°E	N36°E
		Observation	N25°E	(N31°E minimum)	N39°E
Ratio of amplitude between greater and smaller	Calculation		1.0	1.0	1.0
	Observation		1.6	(31)	1.7
Phase angle	greater	Calculation	351°	173°	350°
		Observation	32°	256°	190°
	smaller	Calculation	171°	353°	170°
		Observation	198°	94°	10°
Difference of phase angle between greater and smaller	Calculation		180°	180°	180°
	Observation		166°	(162°)	99°
Derivative of phase angle with respect to azimuth		Observed and Calculated	equal	parallel	parallel

Table XX Summary of relation between observational and theoretical results of pair-observatories.

between Osakayama and Kishu, 70 km between Kishu and Suhara, and 126 km between Suhara and Osakayama. According to Table XIX, there are not large differences among the ratios, which indicates that the differences of observed values of tidal strains are in fairly good conformity with those of theoretical values of strains. By the way, it seems that Suhara has a considerable abnormality from the structural view-point of the earth crust. The author likes to study, in future on this point in minute details.

6. Primary earth tidal strain

As aforesaid, horizontal areal strain due to oceanic tidal load nearly vanishes at these observational stations. Consequently, as the cubical dilatation due to distant tidal load is equal to 0 at earth's surface because there acts no body force in the case of oceanic effect, it is naturally deduced that the vertical strain is also equal to 0. In other expressions,

$$\left. \begin{aligned} e'_{\theta\theta} + e'_{\phi\phi} &= 0 \\ \Delta' = e'_{rr} + e'_{\theta\theta} + e'_{\phi\phi} &= 0 \\ e'_{rr} &= 0 \end{aligned} \right\} \dots\dots\dots(20).$$

Further, by the condition of free surface of the earth, we have at $r=a$

$$e'_{r\phi} = e'_{r\theta} = 0 \dots\dots\dots(21).$$

Therefore, unknown components of strains of oceanic effect are reduced to only two, and they are

$$e'_{\theta\theta} (= -e'_{\phi\phi}) \quad \text{and} \quad e'_{\theta\phi}.$$

Since strain of oceanic effect satisfies the formula (20), the vertical strain, horizontal areal strain and cubical dilatation of direct effect are obtained directly by observed extensions of the ground. The vertical strain is obtained by observation of vertical extension, the horizontal areal strain is obtained as the sum of linear strains in the directions orthogonal to each other along the surface of the earth, and the cubical dilatation is obtained as the sum of vertical and horizontal areal strains.

Since at the earth's surface, by the condition of free surface

$$e_{r\phi} = e_{r\theta} = 0 \dots\dots\dots(22),$$

and the unknowns of components of tidal strain of direct effect are reduced to four, and they are $e_{\theta\theta}$, $e_{\phi\phi}$, $e_{\theta\phi}$ and e_{rr} . As shown in (5) and (6), vertical strain e_{rr} , horizontal areal strain $\Sigma(a)$ and cubical dilatation $\Delta(a)$ at $r=a$ are given by

$$\left. \begin{aligned} e_{rr}(a) &= \frac{1}{ag} V_1 W_2 \\ \Sigma(a) &= \frac{1}{ag} V_2 W_2 \\ \Delta(a) &= \frac{1}{ag} V_3 W_2 \end{aligned} \right\} \dots\dots\dots(23).$$

where

$$\left. \begin{aligned} V_1 &= \frac{g}{a} \left\{ 2a \frac{dF(a)}{dr} + 2F(a) + a^3 \frac{dG(a)}{dr} + 3a^2 G(a) \right\} \\ V_2 &= \frac{2g}{a} \{ a^2 G(a) - F(a) \} \\ V_3 &= ag \left\{ \frac{2}{a} \frac{dF(a)}{dr} + a \frac{dG(a)}{dr} + 5G(a) \right\} \end{aligned} \right\}$$

Using observed values at Osakayama, Kishu and Suhara, horizontal areal strain and cubical strain were obtained, and the numerical values of V_1 , V_2 and V_3

calculated therefrom are shown in Table XXI.

In order to obtain the same accuracy with M_2 -component derived from analysis of one month, it necessitates the period of three months for O_1 , six months for S_2 and one year for K_1 . From this reason, the weight of component tide is assumed to be inversely proportional to their necessary periods of observation.

Observatory	Component Tide	Mean horizontal areal strain		Mean vertical strain		Mean cubical dilatation	
		Amplitude in 10^{-8}	Phase (κ)	Amplitude in 10^{-8}	Phase (κ)	Amplitude in 10^{-8}	Phase (κ)
Osakayama	M_2	2.22	12.3°	0.64	193.6°	1.58	16.5°
"	S_2	0.79	13.5	0.42	163.4	0.42	16.6
"	K_1	1.04	20.1	0.67	189.8	0.39	38.0
"	O_1	1.55	32.8	0.61	231.5	0.98	21.7
Kishu	M_2	1.63	356.9				
Suhara	M_2	2.78	42.8				

Observatory	Component tide	V_2	Weight	V_1	Weight	V_3	Weight
Osakayama	M_2	0.427	24	-0.246	60	0.604	24
"	S_2	0.572	4	-0.615	4	0.616	4
"	K_1	0.294	1	-0.371	2	0.221	1
"	O_1	0.511	4	-0.402	4	0.644	4
Kishu	M_2	0.304	12				
Suhara	M_2	0.521	12				

Table XXI.

From eqs. (8), (23) and Table XXI, we have

$$h-3l=0.434\pm0.026 \quad (\text{at Osakayama, Kishu and Suhara}) \dots\dots\dots(24),$$

and from (23) and Table XXI we get

$$\left. \begin{aligned} 2a \frac{dF(a)}{dr} + 2F(a) + a^3 \frac{dG(a)}{dr} + 3G(a)a^2 &= (-1.82 \pm 0.23) \times 10^5 \\ a^2 G(a) - F(a) &= (2.82 \pm 0.17) \times 10^5 \\ a^2 \left\{ \frac{2}{a} \frac{dF(a)}{dr} + a \frac{dG(a)}{dr} + 5G(a) \right\} &= (3.90 \pm 0.18) \times 10^5 \end{aligned} \right\} \dots\dots(25).$$

From the values of $h-3l$, and $h=0.600\pm0.006$ which is obtained from the tidal tiltmetric observation combined with the observation of Chandler's period of latitude variation, we will obtain

$$l=0.055\pm0.009 \quad \dots\dots\dots(26).$$

Using the value of h , we get from (7) and the second eq. of (9) the follow-

ing relations :

$$\left. \begin{aligned} 2F(a) + a^2G(a) &= (3.903 \pm 0.039) \times 10^6 \\ \frac{dF(a)}{dr} + \frac{2}{a}F(a) + aG(a) &= 0 \end{aligned} \right\} \dots\dots\dots(27).$$

From (25) and (27) we get

$$\left. \begin{aligned} F(a) &= (0.36 \pm 0.06) \times 10^6 \\ a^2G(a) &= (3.18 \pm 0.18) \times 10^6 \\ a \frac{dF(a)}{dr} &= -(3.90 \pm 0.22) \times 10^6 \\ a^3 \frac{dG(a)}{dr} &= -(4.29 \pm 0.97) \times 10^6 \end{aligned} \right\} \dots\dots\dots(28).$$

(c.g.s.)

Using the values of (28), we can easily obtain the value of $dH(a)/dr$ and $dL(a)/dr$ from (7), provided the value of $\partial g/\partial r$ is known. For example, if we assume $g = g_0 + \frac{2g_0(a-r)}{a} \left(1 - \frac{3}{2} \frac{\rho}{\rho_m}\right)$ where ρ (2.67) and ρ_m (5.53) are the density of the surface layer and the mean density of the earth respectively, then we get

$$\left. \begin{aligned} a \frac{dH(a)}{dr} &= -2.58 \pm 0.58 \\ a \frac{dL(a)}{dr} &= -0.69 \pm 0.04 \end{aligned} \right\} \dots\dots\dots(29).$$

Let us now change independent variable from r to ξ by a relation

$$\xi = \frac{r}{a}$$

then, we obtain

$$\left. \begin{aligned} F(\xi) &= (0.030 \pm 0.005) \times \frac{1}{4\pi f a^2} \\ G(\xi) &= (0.267 \pm 0.015) \times \frac{1}{4\pi f a^2} \\ \frac{dF(\xi)}{d\xi} &= (-0.328 \pm 0.018) \times \frac{1}{4\pi f a^2} \\ \frac{dG(\xi)}{d\xi} &= (-0.360 \pm 0.081) \times \frac{1}{4\pi f a^2} \end{aligned} \right\} \dots\dots\dots(30),$$

where f is the gravitation constant.

These observational values almost agree with the results which was calculated theoretically by H. Takeuchi [34] from the earth model derived from the results of velocity distribution of seismic wave and density distribution in the earth's interior. His values are as follows:

$$\left. \begin{aligned} F(\xi) &= 0.044557 \times \frac{1}{4\pi f a^2} \\ G(\xi) &= 0.24159 \times \frac{1}{4\pi f a^2} \\ \frac{dF(\xi)}{d\xi} &= -0.33069 \times \frac{1}{4\pi f a^2} \\ \frac{dG(\xi)}{d\xi} &= -0.29500 \times \frac{1}{4\pi f a^2} \end{aligned} \right\} \dots\dots\dots(31).$$

Further, using observed values of six components of extensions at Osaka-yama, the six strain components at $r=a$ of M_2 -tide will be calculated as follows.

$$\left. \begin{aligned} e_{rr} &= 0.642 \times 10^{-8} \cos(2t - 193.6^\circ) \\ e_{\theta\theta} &= 0.957 \times 10^{-8} \cos(2t - 29.6^\circ) \\ e_{\phi\phi} &= 1.315 \times 10^{-8} \cos(2t - 5.5^\circ) \\ e_{\theta\phi} &= 1.650 \times 10^{-8} \cos(2t - 74.8^\circ) \\ e_{r\theta} &= 0.463 \times 10^{-8} \cos(2t - 226.4^\circ) \\ e_{r\phi} &= 0.539 \times 10^{-8} \cos(2t - 183.3^\circ) \\ \Sigma &= e_{\theta\theta} + e_{\phi\phi} = 2.223 \times 10^{-8} \cos(2t - 12.3^\circ) \\ \Delta &= 1.582 \times 10^{-8} \cos(2t - 16.5^\circ) \end{aligned} \right\} \dots\dots\dots(32).$$

In Figs. 2 (a) and (c) is shown the relation between the amplitude of M_2 -term and their azimuth and in Figs. 2 (b) and (d) the relations between the amplitude and elevation (dip) in the sections of both directions north-south and east-west. The six components of M_2 -term at $r=a$ calculated by the values of (31) which were obtained by H. Takeuchi are reduced as follows.

$$\left. \begin{aligned} e_{rr} &= 0.539 \times 10^{-8} \cos(2t - 180^\circ) \\ e_{\theta\theta} &= 1.085 \times 10^{-8} \cos 2t \\ e_{\phi\phi} &= 0.416 \times 10^{-8} \cos 2t \\ e_{\theta\phi} &= 0.872 \times 10^{-8} \cos 2t \\ e_{r\phi} &= e_{r\theta} = 0 \\ \Sigma &= e_{\theta\theta} + e_{\phi\phi} = 1.504 \times 10^{-8} \cos 2t \\ \Delta &= 0.965 \times 10^{-8} \cos 2t \end{aligned} \right\} \dots\dots\dots(33).$$

(c.g.s.)

In Figs. 4 (a), (b) (c), (d) are shown the relation between amplitudes of M_2 -term and O_1 -term of extension and their azimuth or elevation (dip), calculated by the use of theoretical values (33). According to Fig. 2 (a), the maximum amplitude is seen in the direction of south-east to north-west. Its reason is

obvious, because there are not only the direct effect of the earth tide but also the influence of oceanic tide.

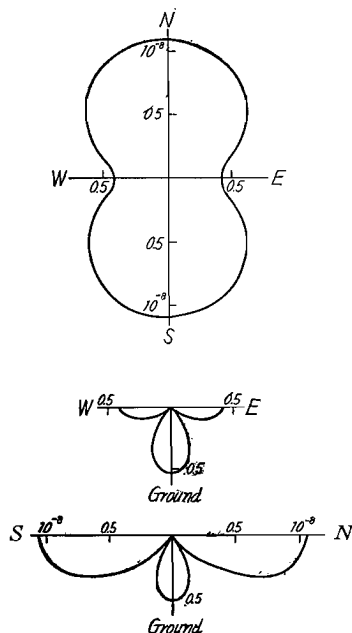


Fig. 4 (a) Relation between the amplitude of linear strain of M_2 -tide and its azimuth or dip obtained by theoretical calculation at Osakayama.

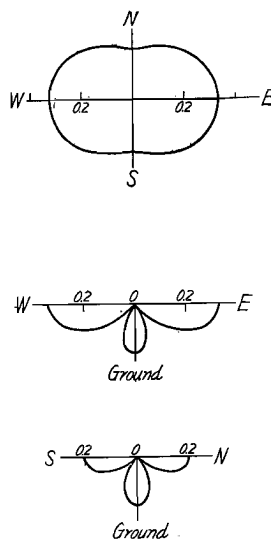


Fig. 4 (b) Relations between the amplitude of linear strain of O_1 -tide and its azimuth or dip obtained by theoretical calculation at Osakayama.

7. Summary

We have observed the tidal strain in six individual directions at Osakayama Observatory, 65 km distant from the nearest sea, and in three horizontal-components at Kishu mine, 15 km distant from the Kumano Sea, and in three horizontal-components at Suhara Observatory on the sea-shore. From these observations, M_2 -terms of the components of tidal strain at these three observation stations were analysed. And a comparison between the relative tidal strains obtained by the differences of observed values at pairs of these stations, and those obtained by means of elastic theory was made. The results obtained by the observations were in a fairly good agreement with the results of theoretical calculations at the pair of Osakayama-Kishu.

The values of vertical strain, horizontal areal strain and cubical dilatation in the primary earth tidal effect were obtained directly from observations.

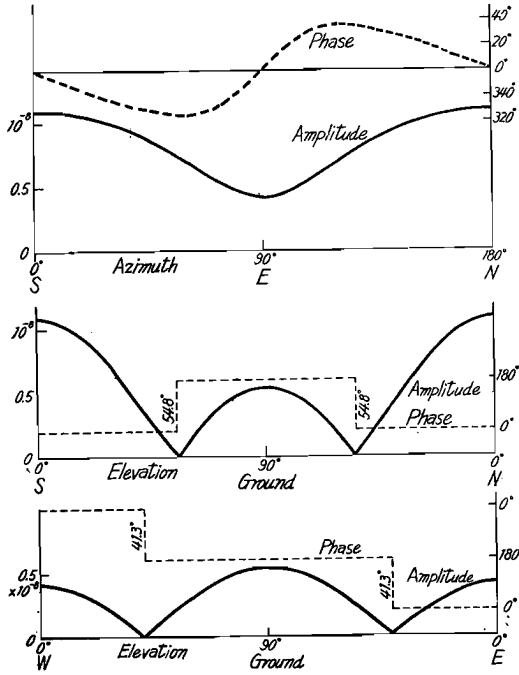
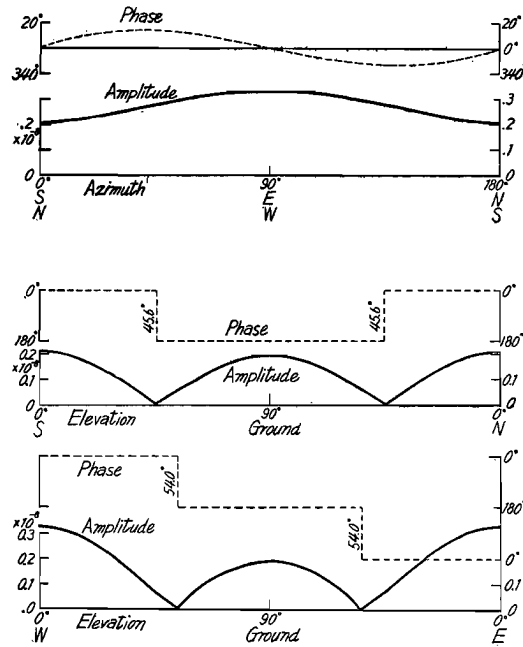


Fig. 4 (c) Theoretical relations between the azimuth or elevation (dip) and the amplitude and phase angle of linear strain of M_2 -term at Osakayama.

Fig. 4 (d) Theoretical relations between the azimuth or elevation (dip) and the amplitude and phase angle of linear strain of O_1 -term at Osakayama.



From these materials the values of l , $F(a)$, $G(a)$, $\frac{dF(a)}{dr}$ and $\frac{dG(a)}{dr}$ at the earth's surface were calculated. These values are in a good agreement with the theoretical values obtained by H. Takeuchi.

As we have seen, both observational and theoretical values showed good agreement in the first approximation, but if we look into minute points, the observational results obtained at Suhara, for instance, showed a considerably large anomalous character compared with those obtained at other two stations, and this local character is supposed to be explained by the complex geological structure near Suhara. The detailed study on the peculiar nature of tidal strain at Suhara will be postponed to near future.

Acknowledgement : The author wishes to express his sincere thanks to Prof. Kenzo Sassa and Prof. Eiichi Nishimura of Kyoto University for their invaluable instructions throughout this study. Many thanks are also extended to Prof. Shoitiro Hayami of Kyoto University who read the whole in manuscript and offered many helpful suggestions.

References

- 1) Sassa, K., Ozawa, I. and Yoshikawa, S. "Observation of tidal strain of the earth." Disaster Prevention Research Institute of Kyoto University, Bulletin 3. (1952) pp. 1-3.
- 2) Shida, T. and Matsuyama, M. : "Change of plumb line referred to the axis of the earth as found from the result of the international latitude observation." Memoir of College of Science and Engineering, Kyoto Imperial University, vol. 4. (1912), pp. 277-284.
- 3) Nishimura, E. : "On the earth tide.", Transactions of the American Geophysical Union, Vol. 31. (1950) pp. 357-376.
- 4) Love, A. E. H. : "The yielding of the earth to disturbing forces.", Proceeding of the Royal Society of London, Vol. 82. (1909) pp. 73-88.
- 5) Chandler, S. C., Astron. Jour., Vol. 11. (1892), 12., 15., 19., 21., 22., 23. (1903).
- 6) Pollack, L. W. ; "Das Periodgramme der Polbewegung", Gerlands Beitrage zur Geophysik, Bd. 16. (1927).
- 7) Lamor, J. The relation of the earth's free precessional nutation to its resistance against tidal deformation", Proceeding of Royal Society of London 82. (1909) pp. 89-96.
- 8) Love, A. E. H. "The gravitational stability of the earth", Philosophical Transactions of the Royal Society of London, A. Vol. 207. (1907), pp. 171-241.
- 9) Hoskins, L. M. H. "The strain of a gravitating compressible elastic sphere", Transactions of the American Mathematical Society, Vol. 21. (1920), pp. 1-43.
- 10) Takeuchi, H. : "Theoretical study of the earth tide", Transactions of the American Geophysical Union, Vol. 31. (1950), pp. 651-689.
- 11) Wadati, K. : "On the travel time of earthquake waves", Geophysical Magazine,

- Vol. 7. (1933), pp. 269-290.
- 12) Gutenberg, B. and Richter, C. F. "On the seismic waves", *Grelands Beitrage zur Geophysik* Bd. 43., (1943) pp. 56-133 ; 45 (1935) pp. 280-360 ; 47. (1937), pp. 73-131.
 - 13) Bullen, K. E. "The variation of density and ellipticities of strata of equal density within the earth", *Monthly Notices of Royal Astronomical Society Geophysical Supplement*, Vol. 3. (1936) pp. 395-401.
 - 14) Shida, T. "On the elasticity of the earth crust", *Memoir of College of Science and Engineering, Kyoto Imperial University*, Vol. 4. (1912) pp. 1-276.
 - 15) Sekiguchi, R. "On the tilting of the earth at Jinsen (Chemulpo) due to tidal load", *Memo. Imp. Marine Obs.*, I. (1922).
 - 16) Takahashi, R. "Tilting motion of the earthcrust caused by tidal loading", *Bulletin of Earthquake Research Institute of Tokyo Imperial University*, Vol. 6., (1929), pp. 85-108.
 - 17) Nishimura, E. ; "On the earth tide", *Tikyubutsuri (Geophysics)*, Kyoto Imperial University, Vol. 5. (1941), pp. 10-86.
 - 18) Corkan, R. H. "A determination of earth tide from tilt observations at two places", *Monthly Notices of the Royal Astronomical Society Geophysical Supplement* Vol. 6. (1953), pp. 431-441.
 - 19) Tomaschek, R. ; "Variations of the tidal vector of gravity at Winsford (Cheshire)", *Monthly Notices of the Royal Astronomical Society Geophysical Supplement* Vol. 6. (1954), pp. 540-556.
 - 20) Hagiwara, T., Rikitake, T., Kasahara, K. and Yamada, J. : "Observation of the deformation of the earth's surface at Aburatsubo, Miura Peninsula. Part III", *Bulletin of the Earthquake Research Institute, Tokyo University*, Vol. 27. (1949), pp. 39-44.
 - 21) Ozawa, I. : "Observation of tidal strain of the earth by extensometer Part II.", *Disaster Prevention Research Institute, Bulletin No. 3.* (1952) pp. 4-17.
 - 22) Boussinesq, J. "Application des Potentials....." Paris, (1885).
 - 23) Terazawa, K. "On the elastic equilibrium of semi-infinite solid", *Journal of College of Science of Tokyo Imperial University* Vol. 37. art. 7. (1916) ; *Philosophical transactions of Royal Society (Ser. A)*, Vol. 217. (1916) pp. 35-50.
 - 24) Nagaoka, H. : "Strains produced by surface loading over a circular area with applications to seismology", *Publ. Earthq. Invest. Commit* 22. B. (1906), 1-15.
 - 25) Lamb, H. : "On Boussinesq's problem", *Proceedings of the London Mathematical Society*, Vol. 34. (1902), pp. 276.
 - 26) Love, A. E. H. "The stress produced in a semi-infinite solid by pressure on part of the boundary", *Philosophical Transactions of the Royal Society of London*, Vol. 228. (1929) pp. 377-420.
 - 27) Sezawa, K. and Nishimura, G. : "Elastic equilibrium of a spherical body under surface tractions of a certain zonal and azimuthal distribution", *Bulletin of the Earthquake Research Institute Tokyo Imperial University*, Vol. 6. (1929), pp. 47-62.
 - 28) Matsumura, M. : "On the settlement of the elastic foundation", *Journal of Civil Engineering Society* Vol. 17. (1931), pp. 813-872.
 - 29) Matsumura, M. "Effect of variation of modulus of elasticity on settlement of elastic foundation", *Journal of Civil Engineering Society* Vol. 17. (1931), pp. 1121-1140.
 - 30) Nishimura, G. "On the deformation of semi-infinite elastic body having a surface

- layer due to the surface loading", Bulletin of the earthquake Research Institute Tokyo University, Vol. 10. (1932) pp. 23-28.
- 31] Ozawa, I. "The observation of tidal variation of the linear strain of the earth", Journal of the geodetic society of Japan, Vol. 2. (1955), pp. 54-58.
- 32] Ogura, S. : "On the tide of the Japanese neighbouring sea", Bulletin of Hydrographic Department Imperial Japanese Navy Vol. VII. (1938).
- 33] Takeuchi, H. : "On the earth tide", Journal of the Faculty of Science University of Tokyo Section II. Vol. 7. (1951), pp. 1-153.
- 34] Kelvin Lord & Tait, P.G. "Treatise on Natural Philosophy. Part. II." (1923).

Appendix

Another method of calculation of the influence of oceanic tide on tidal strain.

Let us suppose that the ground consists of uniform semi-infinite elastic medium. Take a cylindrical coordinate (r, ψ, z) such that r -axis is horizontal on the earth's surface, z -axis vertically inward, and the origin of coordinate at observing point. When a point load P , is applied at the surface $(r, \psi, 0)$, the stress components at observing point are given by

$$\left. \begin{aligned} \widehat{rr} &= \frac{P}{2\pi} \left[(1-2\sigma) \left\{ \frac{1}{r^2} - \frac{z}{r^2} (r^2+z^2)^{-\frac{1}{2}} \right\} - 3r^2z(r^2+z^2)^{-\frac{5}{2}} \right] \\ \widehat{zz} &= -\frac{3P}{2\pi} z^3(r^2+z^2)^{-\frac{5}{2}} \\ \widehat{\psi\psi} &= \frac{P}{2\pi} (1-2\sigma) \left\{ -\frac{1}{r^2} + \frac{z}{r^2} (r^2+z^2)^{-\frac{1}{2}} + z(r^2+z^2)^{-\frac{3}{2}} \right\} \\ \widehat{rz} &= -\frac{3P}{2\pi} rz^2(r^2+z^2)^{-\frac{5}{2}} \\ \widehat{r\psi} &= \widehat{\psi z} = 0 \end{aligned} \right\} \dots\dots\dots(1).$$

The strains and the stresses are connected by the equations

$$\left. \begin{aligned} e'_{rr} &= \frac{1}{E} \{ \widehat{rr} - \sigma(\widehat{\psi\psi} + \widehat{zz}) \} \\ e'_{\psi\psi} &= \frac{1}{E} \{ \widehat{\psi\psi} - \sigma(\widehat{rr} + \widehat{zz}) \} \\ e'_{zz} &= \frac{1}{E} \{ \widehat{zz} - \sigma(\widehat{\psi\psi} + \widehat{rr}) \} \\ e'_{rz} &= \frac{1}{\mu} \widehat{rz} \\ e'_{z\psi} &= e'_{r\psi} = 0 \end{aligned} \right\} \dots\dots\dots(2),$$

where E , μ and σ are Young's modulus, rigidity and Poisson's ratio respectively. From (1) and (2) the components of strain at the ground surface become as follows :

$$\left. \begin{aligned} (e'_{rr})_{z=0} &= \frac{P}{2\pi E} \frac{1}{r^2} (1-\sigma-2\sigma^2) \\ (e'_{\psi\psi})_{z=0} &= -\frac{P}{2\pi E} \frac{1}{r^2} (1-\sigma-2\sigma^2) = -(e'_{rr})_{z=0} \\ (e'_{zz})_{z=0} &= (e'_{r\psi})_{z=0} = (e'_{z\psi})_{z=0} = (e'_{rz})_{z=0} = 0 \end{aligned} \right\} \dots\dots\dots(3).$$

Let cylindrical coordinate (r, ψ, z) transform to rectangular coordinate $(x',$

$y', z')$, and let x' -axis be in the direction of $\psi=0$, y' -axis be in the direction of $\psi = +\frac{\pi}{2}$. Then the formula (3) will be written as follows :

$$\left. \begin{aligned} (e'_{x'x'})_{z'=0} &= \frac{P}{4\pi(\lambda+\mu)} \frac{\cos 2\psi}{r^2} \\ (e'_{y'y'})_{z'=0} &= -\frac{P}{4\pi(\lambda+\mu)} \frac{\cos 2\psi}{r^2} \\ (e'_{x'y'})_{z'=0} &= \frac{P}{4\pi(\lambda+\mu)} \frac{2 \sin 2\psi}{r^2} \end{aligned} \right\} \dots\dots\dots(4),$$

which is the same with the formula (14) in page 8 of this paper.

Publications of the Disaster Prevention Research Institute

The Disaster Prevention Research Institute publishes reports of the research results in the form of bulletins. Publications not out of print may be obtained free of charge upon request to the Director, Disaster Prevention Research Institute, Kyoto University, Kyoto, Japan.

Bulletins :

- No. 1 On the Propagation of Flood Waves by Shoitiro Hayami, 1951.
- No. 2 On the Effect of Sand Storm in Controlling the Mouth of the Kiku River by Tojiro Ishihara and Yuichi Iwagaki, 1952.
- No. 3 Observation of Tidal Strain of the Earth (Part I) by Kenzo Sassa, Izuo Ozawa and Soji Yoshikawa. And Observation of Tidal Strain of the Earth by the Extensometer (Part II) by Izuo Ozawa, 1952.
- No. 4 Earthquake Damages and Elastic Properties of the Ground by Ryo Tanabashi and Hatsuo Ishizaki, 1953.
- No. 5 Some Studies on Beach Erosions by Shoitiro Hayami, Tojiro Ishihara and Yuichi Iwagaki, 1953.
- No. 6 Study on Some Phenomena Foretelling the Occurrence of Destructive Earthquakes by Eiichi Nishimura, 1953.
- No. 7 Vibration Problems of Skyscraper. Destructive Element of Seismic Waves for Structures by Ryo Tanabashi, Takuzi Kobori and Kiyoshi Kaneta, 1954.
- No. 8 Studies on the Failure and the Settlement of Foundations by Sakurō Murayama, 1954.
- No. 9 Experimental Studies on Meteorological Tsunamis Traveling up the Rivers and Canals in Osaka City by Shoitiro Hayami, Katsumasa Yano, Shohei Adachi and Hideaki Kunishi, 1955.
- No.10 Fundamental Studies on the Runoff Analysis by Characteristics by Yuichi Iwagaki, 1955.
- No.11 Fundamental Considerations on the Earthquake Resistant Properties of the Earth Dam by Motohiro Hatanaka, 1955.
- No.12 The Effect of the Moisture Content on the Strength of an Alluvial Clay by Sakurō Murayama, Kōichi Akai and Tōru Shibata, 1955.
- No.13 On Phenomena Forerunning Earthquakes by Kenzo Sassa and Eiichi Nishimura, 1956.
- No.14 A Theoretical Study on Differential Settlements of Structures by Yoshitsura Yokoo and Kunio Yamagata, 1956.
- No.15 Study on Elastic Strain of the Ground in Earth Tides by Izuo Ozawa, 1957.

Bulletin No. 15 Published March, 1957

昭和 32 年 3 月 4 日 印刷

昭和 32 年 3 月 9 日 発行

編輯兼
発行者 京都大学防災研究所

印刷者 山代多三郎

京都市上京區寺之内通小川南入

印刷所 山代印刷株式会社